quiz # 2

Name: ....   ID #:   ....

1. *(10 points)* Find the value of a for which the limit

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - ax}{x^2}$$

is finite and evaluate this limit.

2. *(20 points)* Let $f(x,y) = \ln \left(\dfrac{1}{e - x^2 - y^2} \right)$

   a. find the domain D_f and the range R of f

   b. what's the boundary of D_f, is the domain bounded? Justify.

   c. is the domain closed or open? Justify.

   d. find the equation of the level curve that passes through the point $(0,1)$.

   e. sketch the level curves of f?

3. *(20 points)* Find the area that lies inside the cardioid $r = 2 + 2\cos\theta$ and outside the circle $r = 2$

   (sketch the two curves)

4. *(20 points)* The Fourier series expansion of the function $f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 2 & \pi < x \le 2\pi \end{cases}$ is

$$a_0 + \sum_{n=1}^{+\infty} b_n \sin(nx)$$

   a. find a_0, and b_n

   b. use the series in part a) to show that $\displaystyle\sum_{k=0}^{+\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$.

5. *(7 points)* Find $\displaystyle\lim_{(x,y) \to (0,0)} \frac{x^2 y^2}{\sqrt{x^2 + y^2}}$

6. *(13 points)* Use the two paths test to show that the function $f(x,y) = \dfrac{\ln(1 + xy)}{x^2 + y^2}$ does not have a limit at $(0,0)$